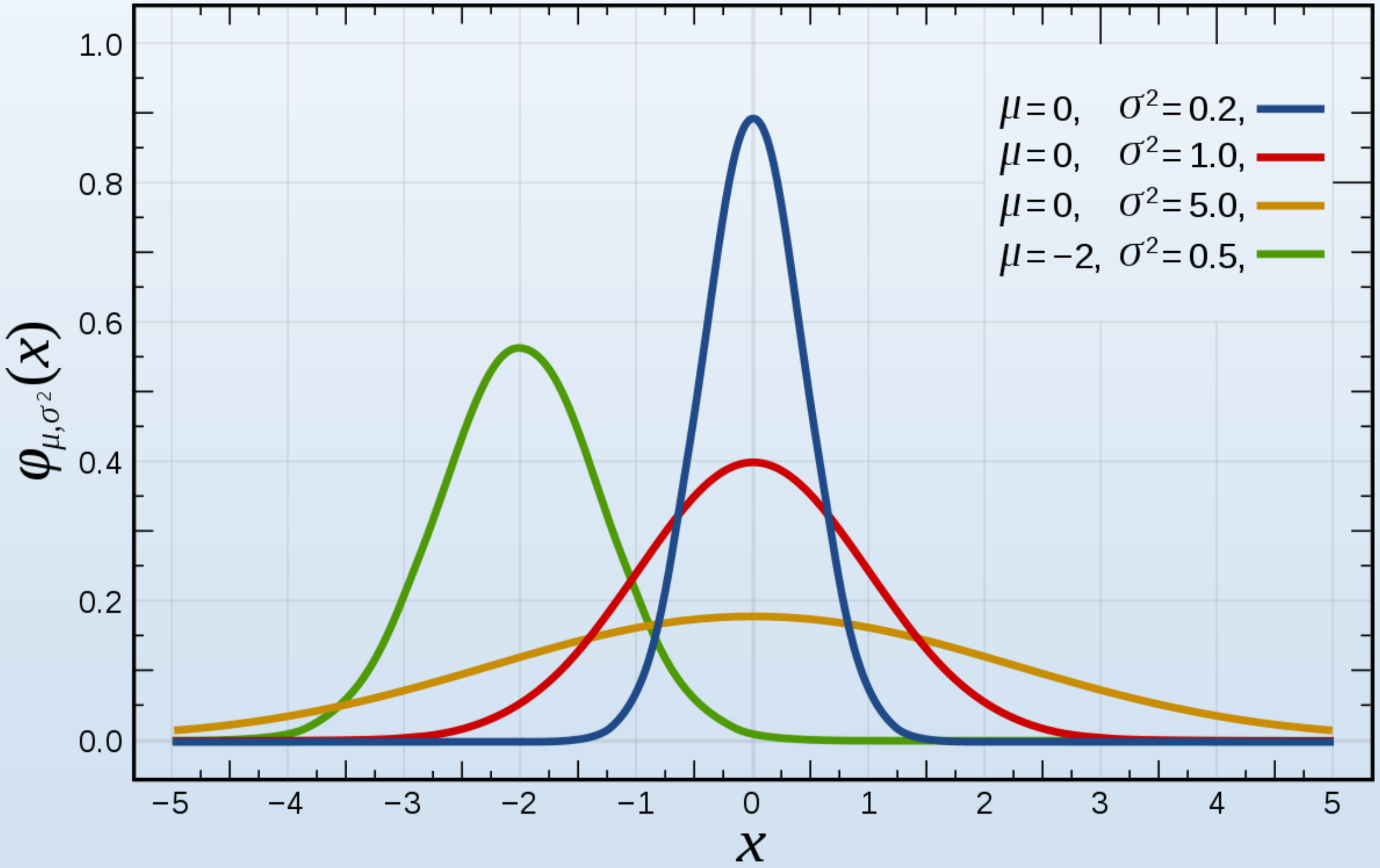
Gaussian/Normal Distribution

- $N(\mu, \sigma)$

Standard Normal Distribution

$N(0,1)$



Interactive website session

https://academo.org/demos/gaussian-distribution/

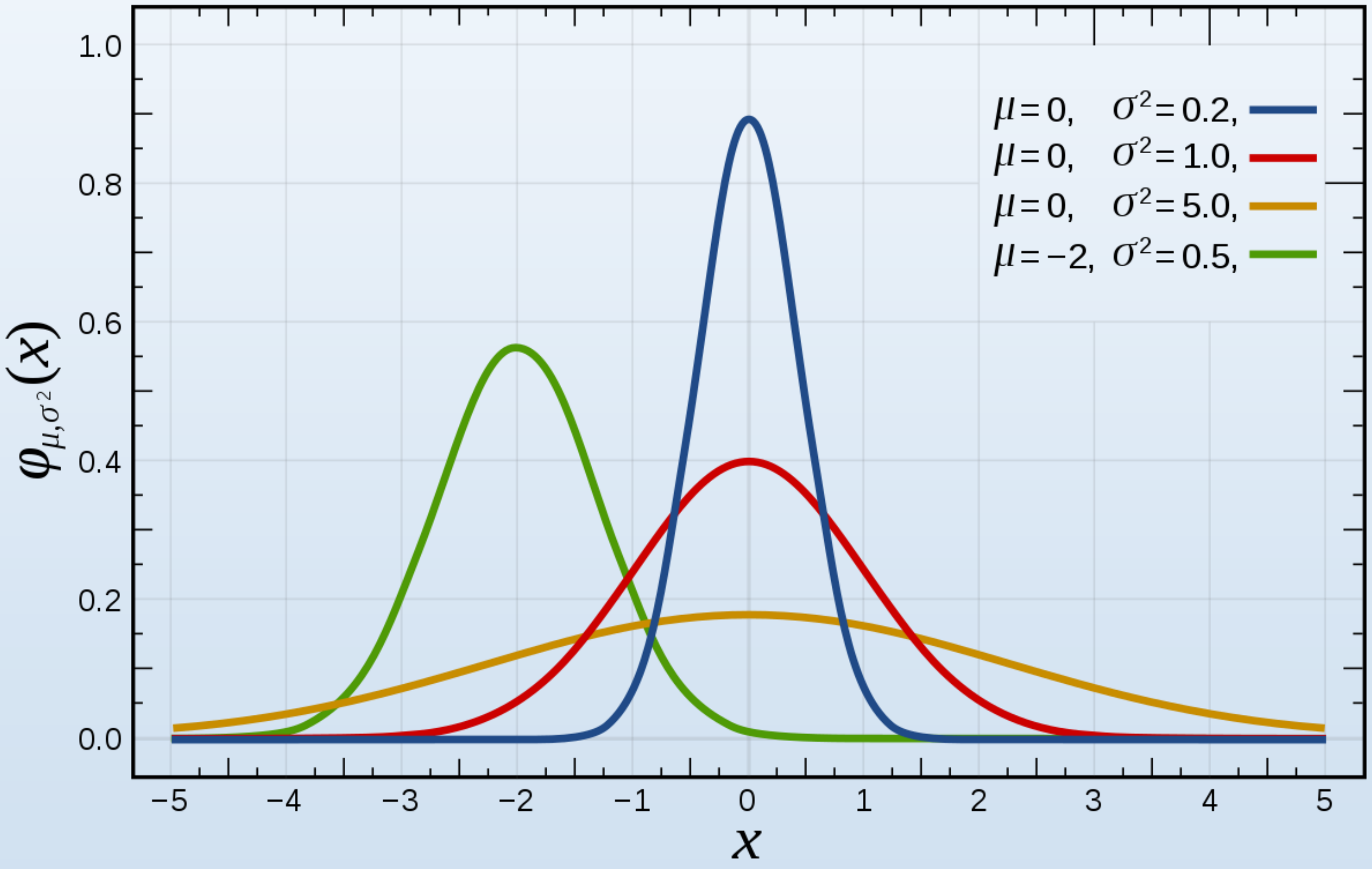
Standard normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$z = \frac{x-\mu}{\sigma/\sqrt{n}}$$

$$t = \frac{x-\mu}{s_1/\sqrt{n}}$$

$$s_1 = \frac{\sum(x_i-\bar{x})^2}{n-1}$$



Numerical Values

- In practical work with normal distribution it is good to remember that about 2/3 of all the values of 'X' to be observed will lie between $\mu \pm \sigma$, about 95 % between $\mu \pm 2\sigma$, and practically all between the three sigma limits $\mu \pm 3\sigma$
- **$P(\mu - \sigma < X \leq \mu + \sigma) \approx 68\%$**
- **$P(\mu - 2\sigma < X \leq \mu + 2\sigma) \approx 95.4\%$**
- **$P(\mu - 3\sigma < X \leq \mu + 3\sigma) \approx 99.7\%$**

- **$P(\mu - 1.96\sigma < X \leq \mu + 1.96\sigma) \approx 95\%$**
- **$P(\mu - 2.58\sigma < X \leq \mu + 2.58\sigma) \approx 99\%$**
- **$P(\mu - 3.29\sigma < X \leq \mu + 3.29\sigma) \approx 99.9\%$**

Example: z-score of an exam mark

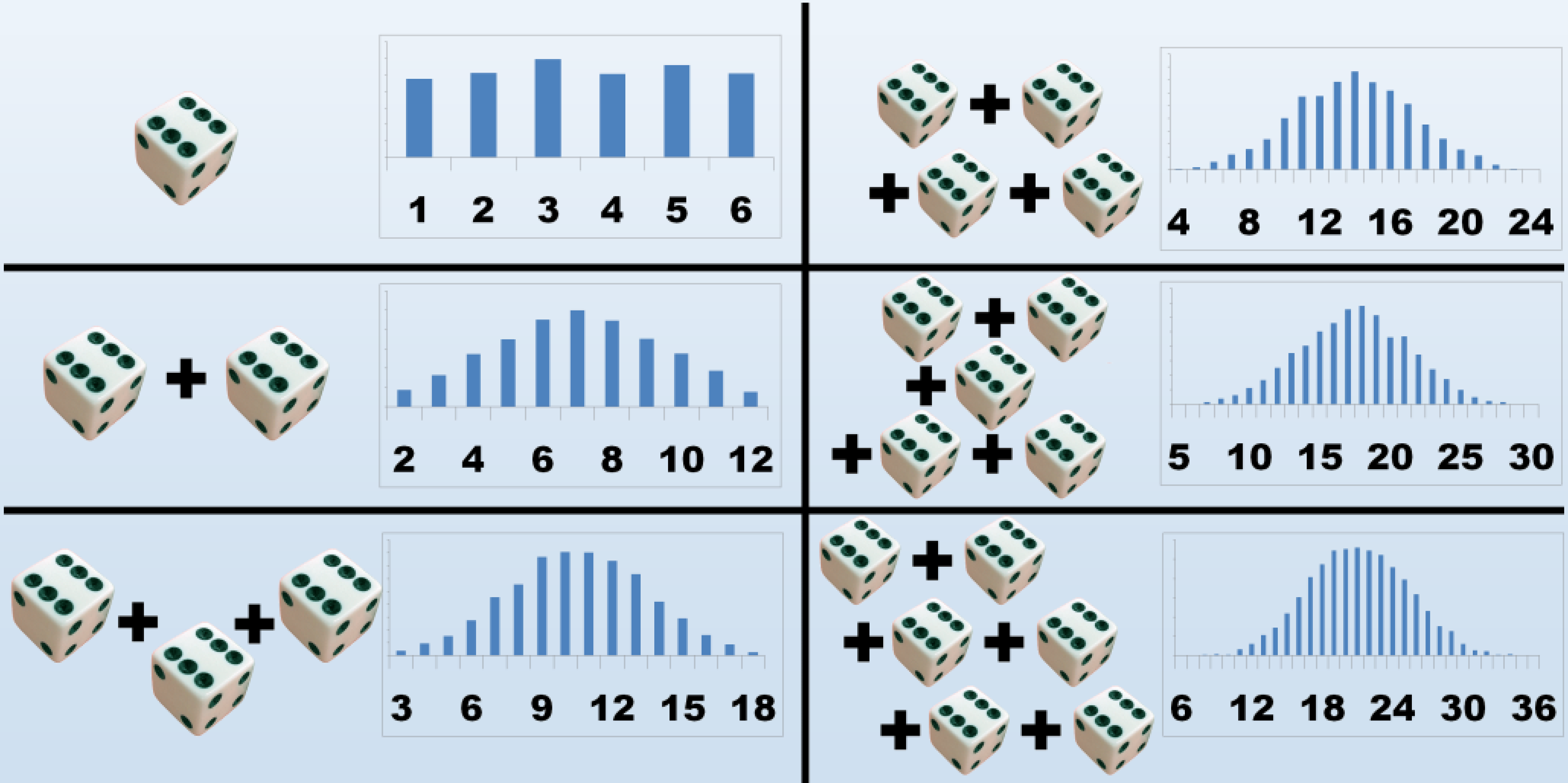
- In BAS-107 (Applied Physics), the class mean is 70 marks with $\sigma = 10$. A student scores 85.

$$z = (85 - 70) / 10 = 1.5$$

- From the z-table, $P(Z \leq 1.5) = 0.9332$ — she has scored better than about 93% of the class.
- Only about 7% of students scored higher. What would $z = -1.5$ mean?

Central Limit Theorem

The Central Limit Theorem states that the sampling distribution of the sample means approaches a normal distribution as the sample size gets larger — no matter what the shape of the population distribution.



CLT in the laboratory

- A single reading of a laser's output power fluctuates — individual readings need not follow any nice distribution.
- By the CLT, the average of n readings is approximately normal, with standard deviation $\sigma/\sqrt{n}$.

Averaging 25 readings shrinks the spread by a factor of 5.

- This is exactly why we repeat and average measurements in the lab — and why error bars shrink slowly: quadrupling the readings only halves the error.

Most projectors offer about 2000 hours of lamp life.

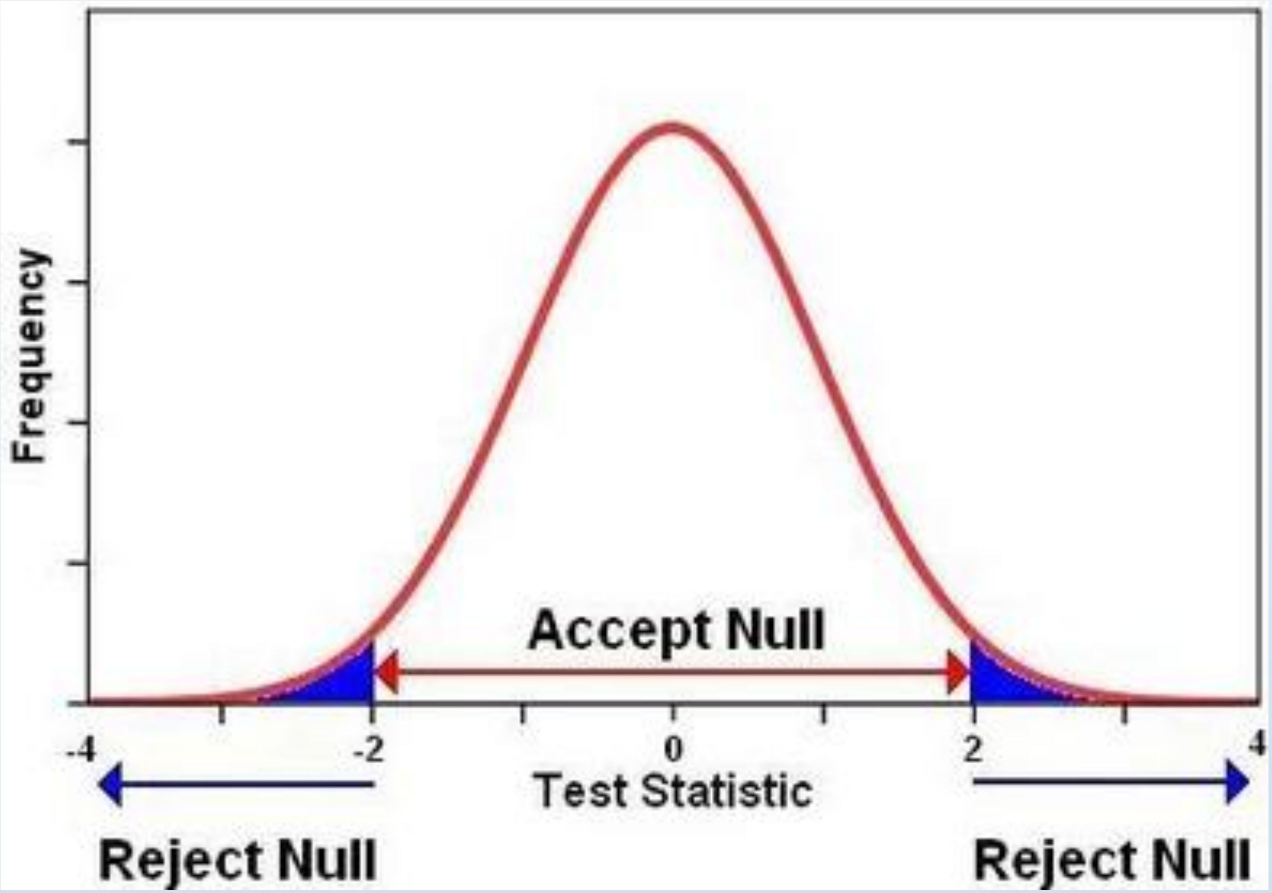
1. Write the above in the form of a hypothesis.
2. Null Hypothesis.
3. Alternate Hypothesis.
4. One tailed test.
5. Two tailed test.

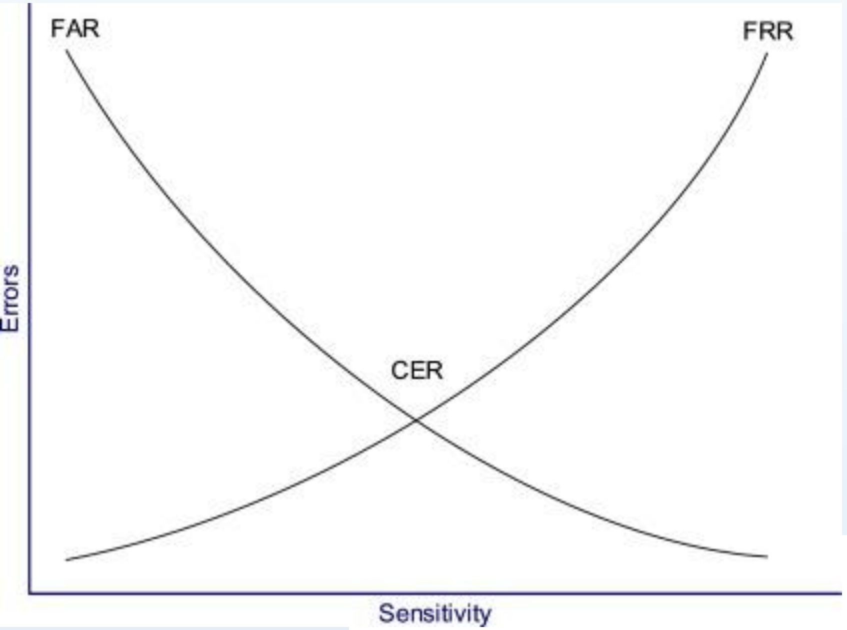
Null Hypothesis

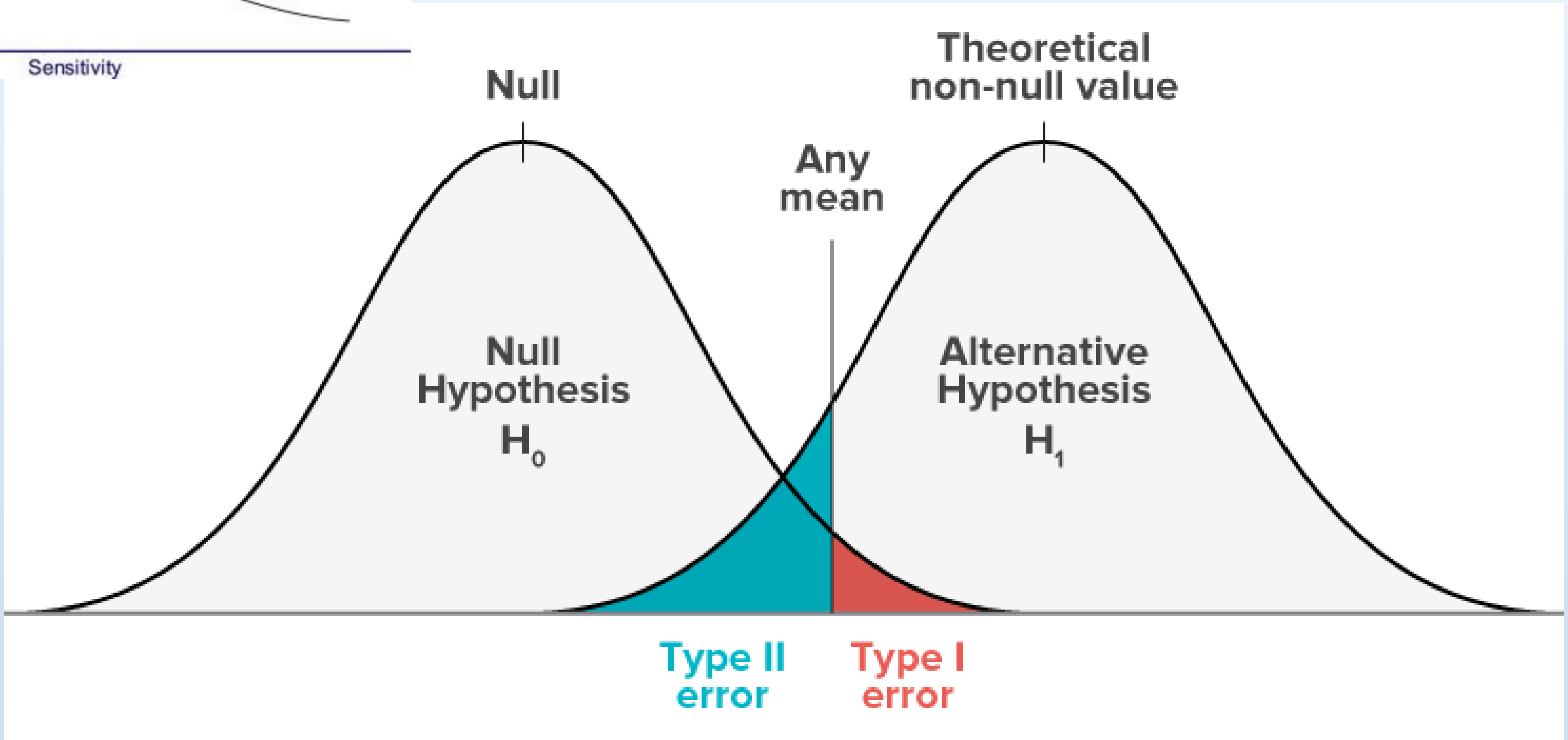
$H_0 : \mu = \mu_0$

Alternate Hypothesis

$H_1 : \mu \neq \mu_0$



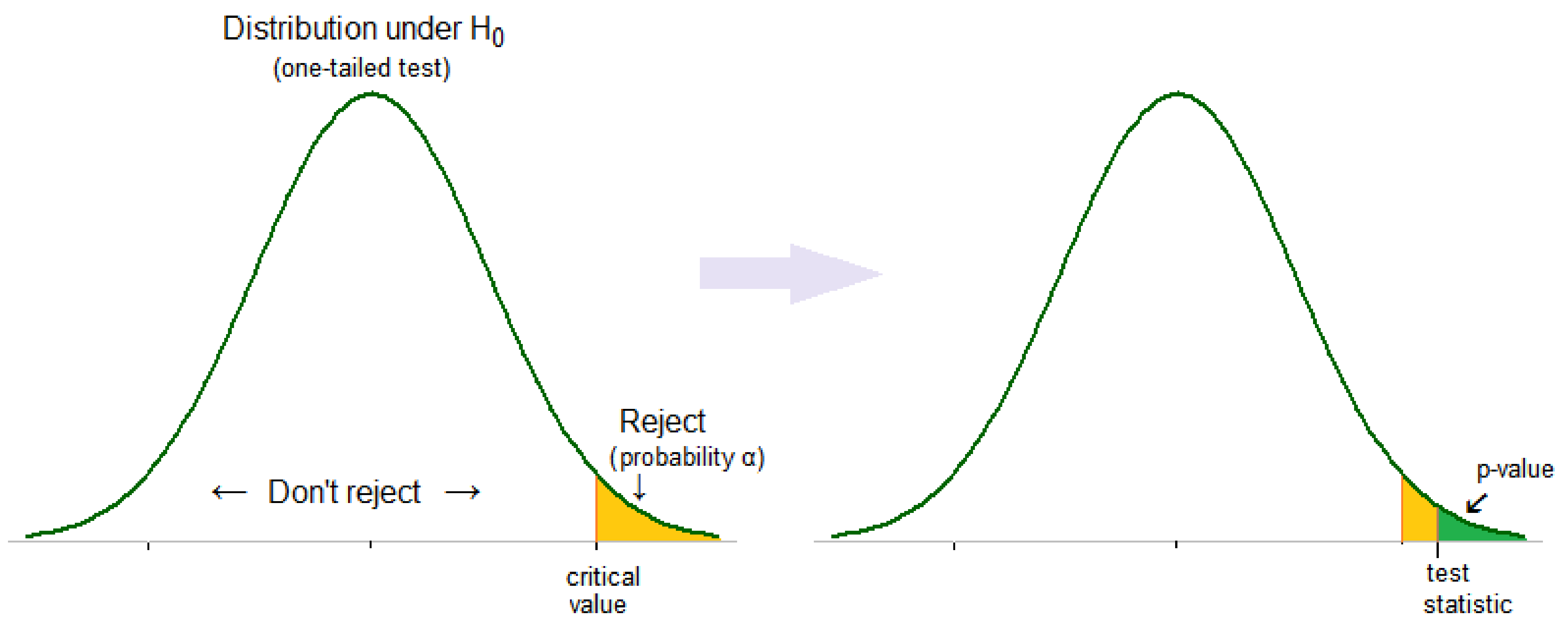

FAR
FRR
Errors
CER
Sensitivity


Theoretical non-null value
Null
Any mean
Null Hypothesis H_0
Alternative Hypothesis H_1
Type II error
Type I error

		Reality	
		True	False
Measured / Perceived	True	Correct ☺	Type I False Positive
	False	Type II False Negative	Correct ☺

Level of significance α

- Maximum probability of committing a Type I error



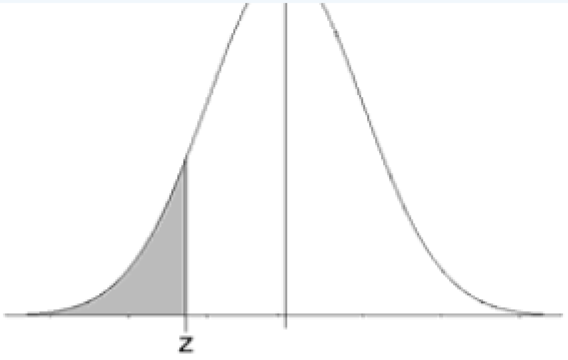
Difference between alpha level and p-value

Alpha sets the standard for how extreme the data must be before we can reject the null hypothesis.

The p-value indicates how extreme the data are.

We compare the p-value with the alpha to determine whether the observed data are statistically significantly different from the null hypothesis.

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

One tailed and two tailed tests

- If testing for whether the coin is biased towards heads, a one-tailed test would be used – only large numbers of heads would be significant.
- However, if testing for whether the coin is biased towards heads or tails, a two-tailed test would be used.

More examples

To figure out if a certain medication can lower the systolic blood pressure.

There's no difference in the mean blood pressure for people that take the placebo compared to people that take the medication.

One and Two tailed tests

- A light bulb manufacturer claims that its' energy-saving light bulbs last an average of 60 days. Set up a hypothesis test to check this claim and comment on what sort of test we need to use

- H_0 : The mean lifetime of an energy-saving light bulb is 60 days.
- H_1 : The mean lifetime of an energy-saving light bulb is not 60 days. (two-tailed)
- H_1 : The mean lifetime of an energy-saving light bulb is less than 60 days. (one-tailed)

Over the years this average has increased or not

- $\boldsymbol{H_0 \colon \mu \leq \mu_0}$
- $\boldsymbol{H_1 \colon \mu > \mu_0}$

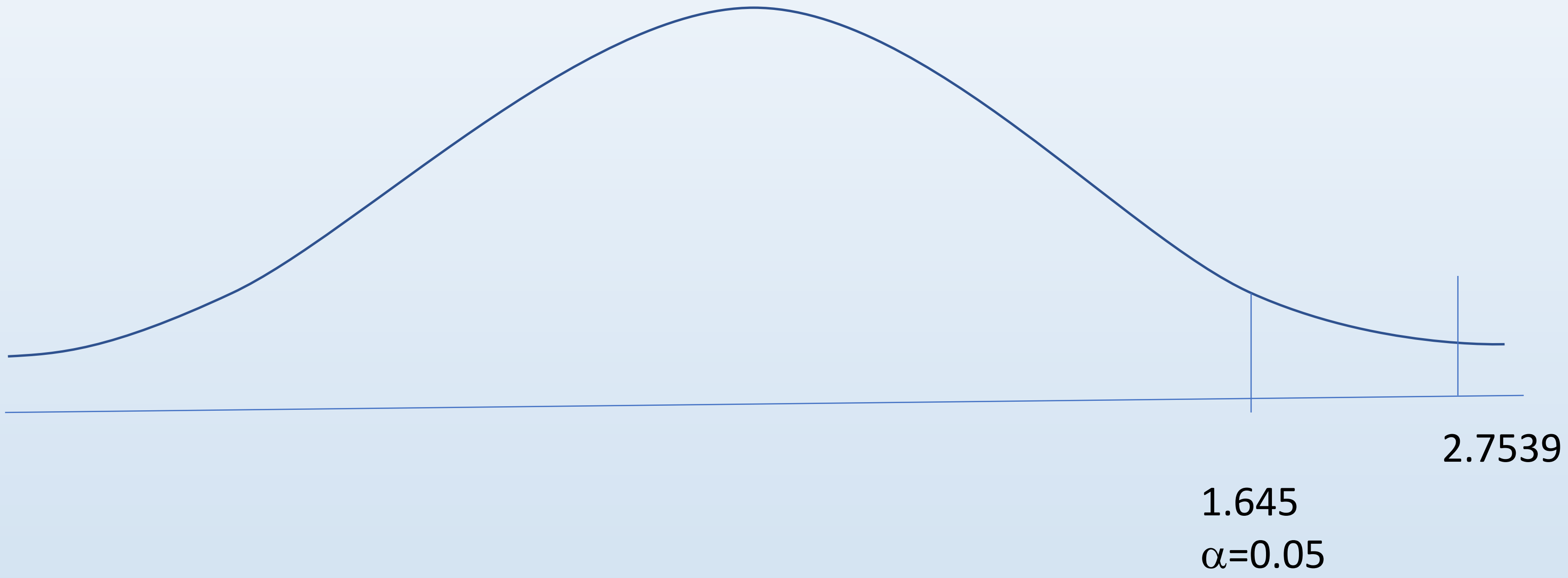
- We reject the $\boldsymbol{H_0}$ in the favour of $\boldsymbol{H_1}$

Example (10.1 p187)

- A survey , done 10 years ago , of CPA's found that their average salary was \$74,914/-. An accounting researcher would like to test whether over the years this average has increased or not? A sample of 112 CPAs produced a mean salary of \$78,695. Also given that the population standard deviation σ = \$14,530.

- $\boldsymbol{Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$

$$Z = 2.7539$$



- Computed value > critical value at 5% level of significance, we conclude that the average salary of CPA has increased.

- $\boldsymbol{H_o: \mu = \mu_o}$

- Since the computed z (2.7539) exceeds the critical value (1.645 at α = 0.05), we reject the null hypothesis that the salary has remained the same.

Interval Estimation

Let θ be the unknown parameter and let T be the unbiased point estimate E(T)= θ

Now, fix a desired level of confidence denoted as

(1-α) $\times$ 100 % . Here 'α' is the probability of type –I errors.

Usually , confidence level is fixed as 90%, 95 %, or 99%.

Thus , for 95 % confidence level α = 0.05.

The interval estimate of θ is T-h, T+h

$$P(T - h \leq \Theta \leq T + h) = 1 - \alpha$$

Parametric and non-parametric tests

- If the information about the population is completely known by means of its parameters, the statistical test is called a parametric test
- Eg: t-test, f-test, z-test, ANOVA

- If there is no knowledge about the population or its parameters, but we still need to test a hypothesis about the population, it is called a non-parametric test.
- Eg: Mann–Whitney, Wilcoxon rank-sum