Maxwell equations in differential form

Gauss's law for electricity

$$\oint \boldsymbol{D} \cdot \boldsymbol{dS} = \boldsymbol{q_{enclosed}}$$

$$\oint \boldsymbol{D} \cdot \boldsymbol{dS} = \int_{\boldsymbol{V}} \rho dV$$

$\oint \boldsymbol{A} \cdot \boldsymbol{dS} = \int \boldsymbol{\nabla} \cdot \boldsymbol{A} dV$ Divergence theorem

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho$$

Gauss's law for magnetism

$$\oint \boldsymbol{B} \cdot \boldsymbol{dS} = \boldsymbol{0}$$

$$\int \boldsymbol{\nabla} \cdot \boldsymbol{B} dV = 0 \text{ Using Divergence theorem}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \boldsymbol{0}$$

Faraday's law

$$\varepsilon = -\frac{d\phi_B}{dt}$$

(here ϕ_B denotes the Magnetic flux)

$$\oint \boldsymbol{E} \cdot \boldsymbol{dl} = -\int_{\boldsymbol{S}} \frac{\partial \boldsymbol{B}}{\partial \mathrm{t}} \cdot \boldsymbol{dS}$$

Stokes theorem $\oint \boldsymbol{A} \cdot \boldsymbol{dl} = \int_{\boldsymbol{S}} \boldsymbol{\nabla} \times \mathbf{A} \cdot \boldsymbol{dS}$

$$\int_{\boldsymbol{S}} \boldsymbol{\nabla} \times \mathbf{E} \cdot \boldsymbol{dS} = -\int_{\boldsymbol{S}} \frac{\partial \boldsymbol{B}}{\partial \mathrm{t}} \cdot \boldsymbol{dS}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathrm{t}}$$

Ampere's Law

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = \mathrm{I}$$

Maxwell's addition to Ampere's Law

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = \mathrm{I} + \frac{d\phi_D}{dt}$$

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = \mathrm{I} + \int_{\boldsymbol{S}} \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}} \cdot \boldsymbol{dS}$$

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = \int_{\boldsymbol{S}} \boldsymbol{J} \cdot \boldsymbol{dS} + \int_{\boldsymbol{S}} \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}} \cdot \boldsymbol{dS}$$

$\int_{\boldsymbol{S}} \boldsymbol{\nabla} \times \mathbf{H} \cdot \boldsymbol{dS} = \int_{\boldsymbol{S}} \boldsymbol{J} \cdot \boldsymbol{dS} + \int_{\boldsymbol{S}} \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}} \cdot \boldsymbol{dS}$ **Applying Stokes theorem**

$$\boldsymbol{\nabla} \times \mathbf{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}}$$

Summary

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathrm{t}}$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial \mathrm{t}}$$

For linear medium

$\boldsymbol{B} = \mu \boldsymbol{H}, \mathbf{J} = \sigma \mathbf{E}, \mathbf{D} = \boldsymbol{\epsilon} \mathbf{E}$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

Website

https://phet.colorado.edu/sims/html/faradays-law/latest/faradays-law_all.html