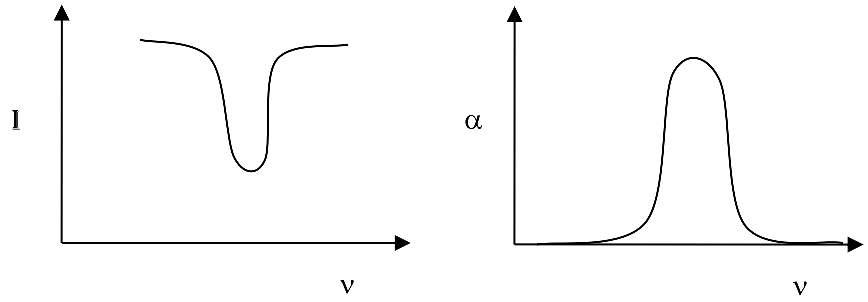
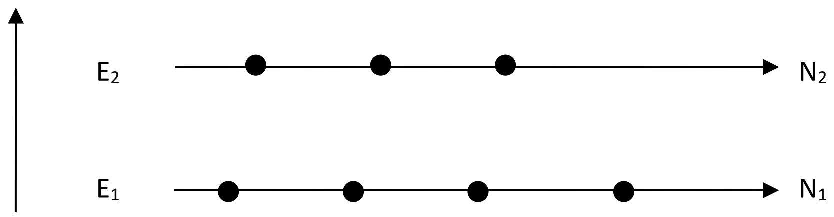
Principle of laser, stimulated and spontaneous emission

For an absorbing medium the intensity gets attenuated with distance. The attenuation is a function of frequency of the incoming wave

$$I = I_0 e^{-\alpha x}$$



The number of atoms per unit volume that occupy a given energy level is called the **population** of that energy level.



Let the populations at level E_1 be N_1 and at E_2 be N_2 . At thermal equilibrium the population at the energy levels can be found from Boltzmann law

$$N_1 = e^{-\frac{E_1}{kT}}$$

$$N_2 = e^{-\frac{E_2}{kT}}$$

The relative population is given by

$$\frac{N_2}{N_1} = e^{-\frac{E_2-E_1}{kT}}$$

When a photon travels through a medium three different processes are likely to occur. They are absorption, spontaneous emission and stimulated emission.

Absorption

$$R_{abs} = -\frac{dN_1}{dt} = \frac{dN_2}{dt}$$

Rate of absorption transition can also be written as

$$R_{abs} = B_{12}\rho(\nu)N_1$$

Where N_1 is population of atoms at E_1, $\rho(\nu)$ is the **energy density of the incident beam** and B_{12} is the constant of proportionality also known as Einstein coefficient.

Spontaneous Emission

An atom cannot stay in an excited state for a longer duration. In a time of about 10^{-8}s , the atom reverts to the lower energy state by releasing a photon. Emission of a photon by an atom without any external impetus is called **spontaneous emission**.

$$R_{sp} = A_{21}N_2$$

Stimulated Emission

Electrons may make a downward transition with the interaction with photons.

$$R_{st} = B_{21}\rho(\nu)N_2$$

In stimulated emission each incident photon encounters a previously excited atom, and the optical field of the photon interacts with the electron. The result of the interaction is a kind of resonance effect, which induces each atom to emit a second photon with the same frequency, direction, phase and polarization as the incident photon.

Einstein relations

Under thermal equilibrium the mean population in the lower and upper energy levels should remain constant.

$$+B_{12}\rho(\nu)N_1 = A_{21}N_2 + B_{21}\rho(\nu)N_2$$

$$\rho(\nu) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$\rho(\nu) = \frac{A_{21}/B_{12}}{\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}}$$

As

$$\frac{N_2}{N_1} = e^{-\frac{E_2 - E_1}{kT}} = e^{-\frac{h\nu}{kT}}$$

$$\rho(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{e^{\frac{h\nu}{kT}} - \frac{B_{21}}{B_{12}}}$$

To maintain thermal equilibrium the system must release energy in the form of electromagnetic radiation. It is required that the radiation be identical with black body radiation and be consistent with Planck's radiation for any value of T.

According to Planck's law

$$\rho(\nu) = \frac{8\pi h\nu^3\mu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Where $\mu$the refractive index of the medium and c is is the velocity of light in free space.

Thus $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3\mu^3}{c^3}$ and $B_{21} = B_{12}$.

$$B_{21} = B_{12} = \frac{c^3}{8\pi h\nu^3\mu^3}$$

The above relations are known as Einstein relations and the coefficients are known as Einstein coefficients. The relation also shows that the ratio of coefficients of spontaneous verses stimulated emissions proportion the third power of frequency of the radiation. This is why it is difficult to achieve laser action in higher frequency ranges such as x-rays.