Plane electromagnetic wave propagation

Consider a uniform but source free medium having dielectric constant ϵ, magnetic permeability μ, and conductivity σ

Uniform medium

Linear : $\mathbf{D}$ is parallel to **E**

: $\mathbf{B}$ is parallel to **H**

Homogeneous : Medium properties are same at all points

Isotropic : μ and ϵ are independent of directions (μ and ϵ are scalar constants)

Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \mathbf{H}}{\partial \mathrm{t}}$$

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \epsilon \frac{\partial \mathbf{E}}{\partial \mathrm{t}}$$

$$\nabla \cdot \boldsymbol{H} = 0$$

$$\nabla \cdot \boldsymbol{E} = 0$$

Writing the above four coupled equations in terms of **E**

$$\nabla \times \left(\nabla \times \boldsymbol{E} = -\mu \frac{\partial \mathbf{H}}{\partial \mathrm{t}}\right)$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \left(-\mu \frac{\partial \nabla \times \mathbf{H}}{\partial \mathrm{t}}\right)$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \left(-\mu \frac{\partial}{\partial \mathrm{t}}\left(\sigma \boldsymbol{E} + \epsilon \frac{\partial \mathbf{E}}{\partial \mathrm{t}}\right)\right)$$

Identity

$$\mathbf{A} \times (\mathbf{B} \times \boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \nabla(\nabla \cdot \boldsymbol{E}) - \boldsymbol{E}(\nabla \cdot \nabla)$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = -\nabla^2 \boldsymbol{E}$$

$$-\nabla^2 \boldsymbol{E} = -\mu\sigma \frac{\partial \boldsymbol{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \boldsymbol{E} = \mu\sigma \frac{\partial \boldsymbol{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Similarly

$$\nabla^2 \boldsymbol{H} = \mu\sigma \frac{\partial \boldsymbol{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

These are known as general wave equations

These equations govern the behavior of an electromagnetic field in uniform but source free conducting medium. The presence of first order term $\frac{\partial}{\partial t}$ indicates, the fields decay as they propagate through the medium. For this reason, a conducting medium is called a lossy medium.

Plane wave in dielectric medium

Here $\sigma = 0$

$$\nabla^2 \boldsymbol{E} - \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\nabla^2 \boldsymbol{H} - \mu\epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

These are also known as time dependent Helmholtz equations.

The absence of first order term indicates that electromagnetic fields do not decay as they propagate in a lossless medium.