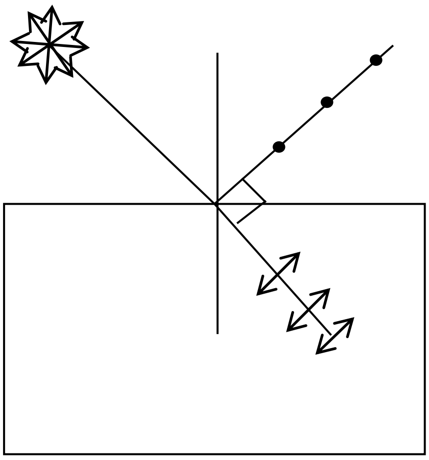
Polarization

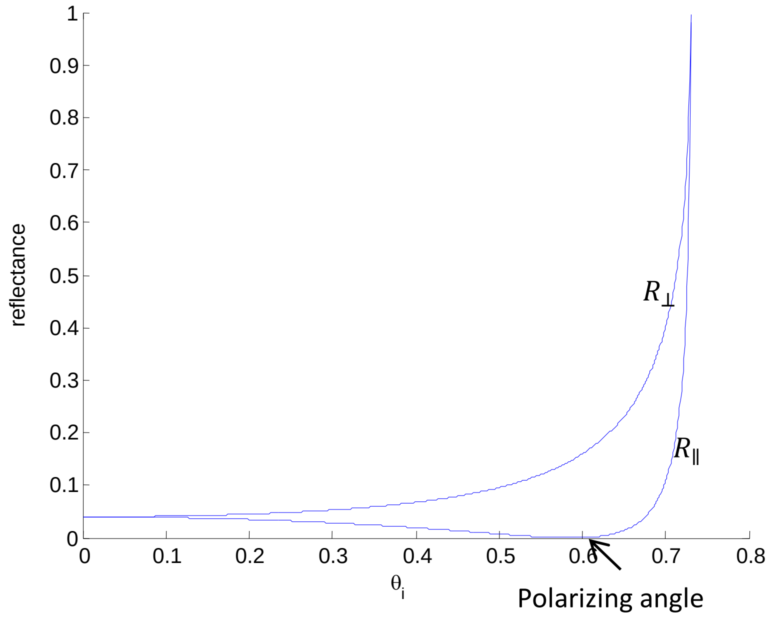
Production of polarized light by various methods



Polarization by reflection

The incident electric field can be resolved into two components. One perpendicular to the plane of incidence (p- component) and another lying in the plane of incidence called the s-component.

Light reflected from the surface of transparent media is partially polarized. The percentage of polarized reflected beam is greatest at an incident angle known as angle of polarization (also known as Brewster Angle). For ordinary glass Brewster angle = 57.5^0.



This method is not very advantageous. Only 15% of the s-component is reflected.

Variation of reflectance of perpendicular and parallel component is shown above.

Explanation for reflectance curve

$$R_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel}^{2} = \frac{tan^2(\theta_i - \theta_t)}{tan^2(\theta_i + \theta_t)}$$

$$R_{\perp} = \left(\frac{E_{or}}{E_{oi}}\right)_{\perp}^{2} = \frac{sin^2(\theta_i - \theta_t)}{sin^2(\theta_i + \theta_t)}$$

$R_{\perp}$ can never be zero.

$R_{\parallel}$ will be zero when $\theta_i + \theta_t = 90°$

Brewster's Angle

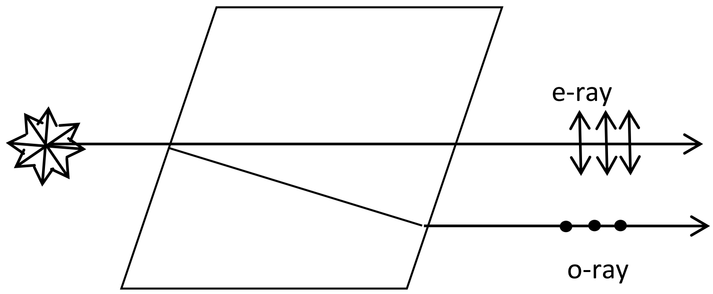
Maximum polarization of the reflected ray occurs when it is at right angles to the refracted ray.

$$\mu = \frac{sin(\theta_p)}{sin(90 - \theta_p)}$$

$$\mu = tan(\theta_p)$$

Brewster windows are used in gas lasers.

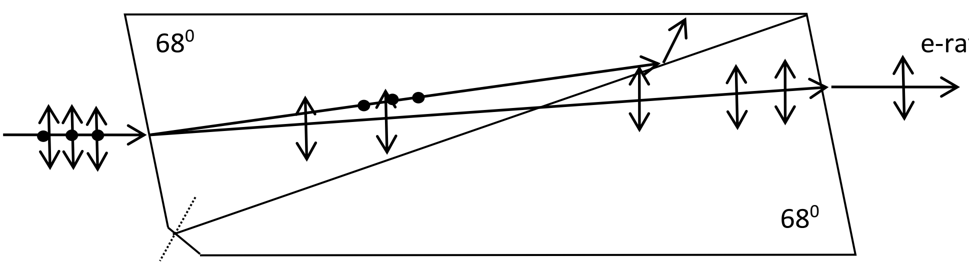
Polarization by double refraction



The phenomena causing two refracted rays by a crystal is called birefringence. The two rays produced are linearly polarized in mutually polarized direction. One ray obeys Snell's law and is called the ordinary ray and other ray does not obey the snell's law and is called the extraordinary ray.

Nicol Prism

A suitably long, narrow calcite rhobohedron, after cutting the rhomb diagonally, the two pieces are polished and cemented back together with Canada Balsam. Balsam cement is transparent and has refractive index 1.55 (between n_e and n_o). The critical angle at the Balsam interface is about 69^0. The o-ray will be totally internally reflected and therefore absorbed by a layer of black paint at the side of the rhomb. The e-ray emerges laterally displaced (Canada Balsam absorbs UV).



Polaroid Sheets

A clear plastic sheet of long chain molecules of polyvinyl alcohol is heated and then stretched in a given direction to many times its original length. During the stretching process the polyvinyl alcohol molecule becomes aligned along the direction of stretching . The sheet is then laminated to a rigid sheet of plastic to stabilize its size. It is then exposed to iodine vapour. The iodine atoms attach themselves to the straight line chain poly vinyl molecules and consequently form long parallel conducing chains. When natural light is incident on the sheet the component that is parallel to the chain of the iodine atoms induces current in the conducting chains and is therefore strongly absorbed. Consequently, the light transmitted contains only the component that is perpendicular to the direction of molecular chains. The direction of the electric field vector in the transmitted beam corresponds to the transmission axis.

Quarter and Half Wave Plates

A plate cut from a *doubly refracting uniaxial crystal* , with its optic axis parallel to the refracting faces can be used to introduce a given phase difference between o and e waves travelling normally through it. They are used to change linear to elliptical polarization or vice versa.

QWP: Phase difference: $\frac{\pi}{2}$ Path difference: $\frac{\lambda}{4}$

HWP: Phase difference: π Path difference: $\frac{\lambda}{2}$

If ‘d’ is the thickness of the plate then the resultant path difference between the two emerging waves will be

$$(\mu_0 - \mu_e)d$$

Thus the corresponding phase change will be

$$\frac{2\pi}{\lambda}(\mu_0 - \mu_e)d$$

Generally d is chosen such that the path difference is $n\lambda + \frac{\lambda}{4}$, or $n\lambda + \frac{\lambda}{2}$, otherwise the wave plate will be very fragile.

Optically active materials

Plane of vibration of a beam of linearly polarized light undergoes a continuous rotation as it propagates along the optically active medium.

Dextrorotatory	right circular rotation
Levorotatory	left circular rotation
Quartz	$21.7°/mm$ for Na light
HgS (Cinnabar)	$32.5°/mm$ for Na light
Turpentine	$-37°/10mm$ at 10°C $\lambda = 589.3nm$

Dichoric crystals

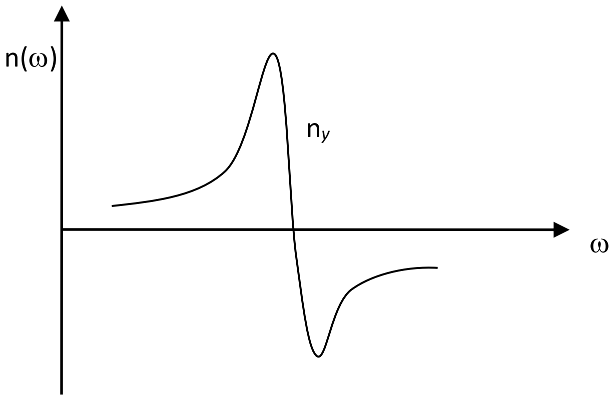
Due to anisotropy in their crystalline structure, electric field component of incident light wave is strongly absorbed in certain direction. Electrons associated with a given atom are also under the influence of the surrounding nearby atoms, which themselves may not be symmetrically distributed. As a result, the elastic binding forces on the electrons will be different in different directions.

If in addition to anisotropic, the material is absorbing, and the incoming light is white, the crystal will appear colored, and the color will depend upon orientation.

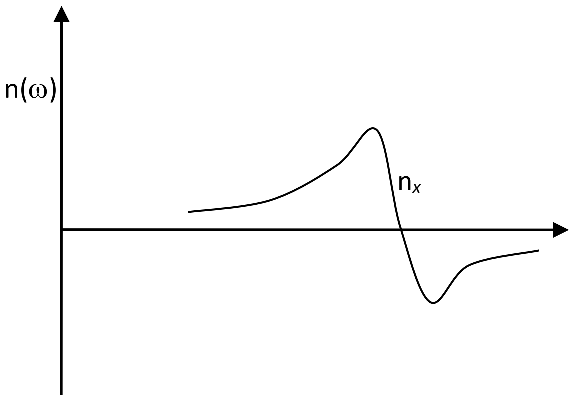
Substances which exhibit two colors are known as dichoric.

Birefringence

Crystals belonging to hexagonal, tetragonal and trigonal systems have their atoms arranged so that light propagating in some general direction will encounter an asymmetric structure.



Put a dot on a sheet of paper and view it through calcite crystal. Two images will be seen. On rotating the crystal about the incident beam as axis, one image remains stationary while the other rotates around the first.



The stationary image is known as ordinary image and the rotating image is known as extraordinary image. A material which displays two different refractive indices is known as birefringent.

$\frac{dn}{d\omega} < 0$ absorption band.

$$n_0 = \frac{c}{v_\perp}$$

$$n_e = \frac{c}{v_\parallel}$$

$\Delta n = n_e - n_0$ called birefringence.

Calcite $\Delta n = n_e - n_0 = -0.172$

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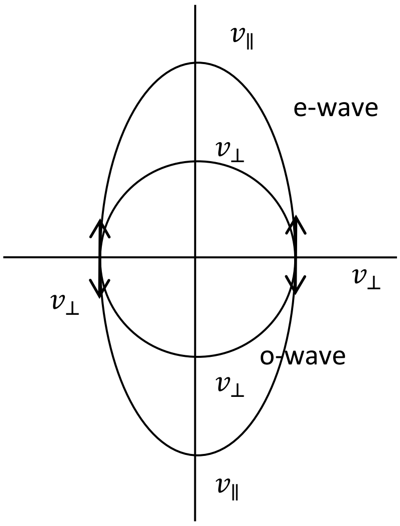
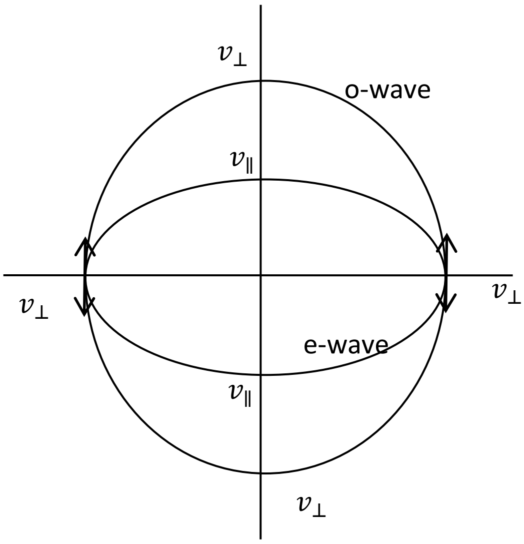
Quartz $\Delta n = n_e - n_0 = +$





Specific rotation

If an optically active material is kept between two crossed polarizers, the field of view becomes bright. In order to get darkness once again, the analyzer has to be rotated through an angle. The angle through which the analyzer is rotated equals the angle through which the place of polarization is rotated by the optically active substance.

The property of a substance of rotating the plane of polarization about the direction of propagation of light is called optical activity and the substances having this property are called optically active substances.

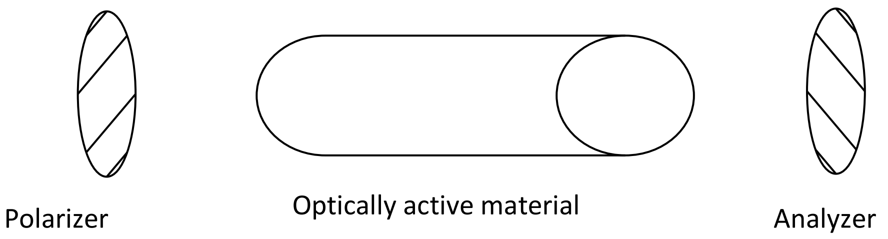
Some substances rotate the plane clockwise called right handed or dextrorotatory while others doing so in anticlockwise direction are called left handed or laevorotatory. Cane sugar is dextrorotatory while the fruit sugar is laevorotatory.

Biot's law of optical activity

The angle of rotation is directly proportional to the thickness of the substance transversed, concentration of the solution, and inversely proportional to the square of the wavelength passing through it.

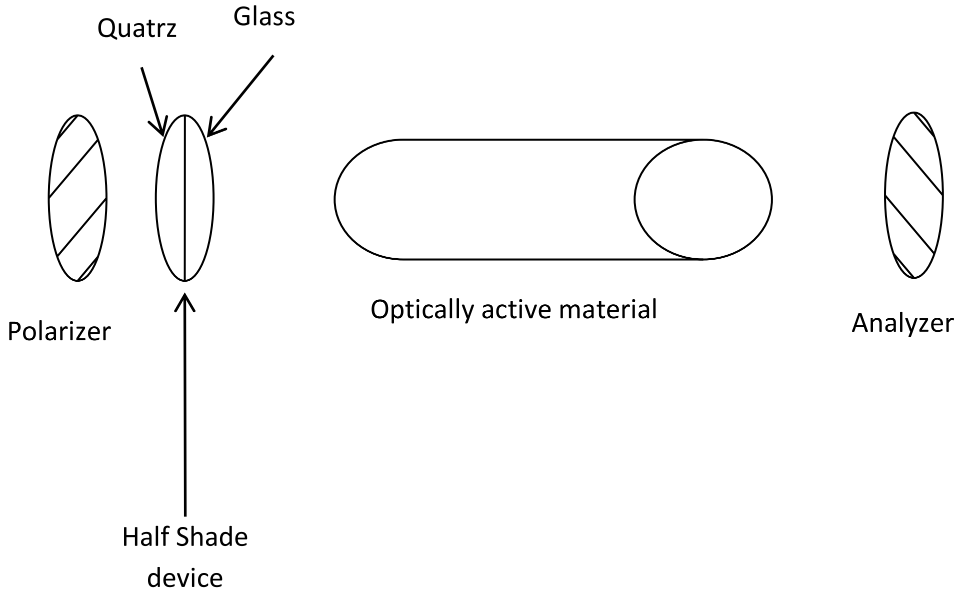
$$S = \frac{\theta}{l \cdot c}$$

S known as specific rotation, l length in decimeter, c concentration in g/cc



Laurent half shade device

It is a device used to accurately adjust the position of pair of polarizer and analyzer for crossed position. It consists of two semi-circular plates, one is made of uniaxial (calcite/quartz) crystal and the other is a glass plate. The portion of which is made of calcite, is so cut that t its optic axis, is parallel to its faces and the thickness is just sufficient to produce a path difference $\lambda/_2$ between e- and o – rays (hence the half-wave-plate). The two halves of the plate are thus unequally illuminated. The analyzer is rotated unless the field of view becomes equally dark. Such equal illuminations positions(angles) of analyzer serve as a reference to determine the specific rotations of optically active materials.



Polarized light

Light for which the orientation of the electric field is constant although its magnitude and sign vary in time.

Imagine two harmonic, linearly polarized light waves of same frequency, moving through same region of space, in the same direction. Their respective electric fields directions are mutually perpendicular.

$$E_y = E_{y0} cos(kx - \omega t)$$

$$E_z = E_{z0} cos(kx - \omega t + \delta)$$

$$E_z = E_{z0} cos(kx - \omega t) cos(\delta) - E_{z0} sin(kx - \omega t) sin(\delta)$$

$$E_z = E_{z0} cos(kx - \omega t) cos(\delta) - E_{z0} \sqrt{1 - cos^2(kx - \omega t)} sin(\delta)$$

$$E_z = E_{z0} \frac{E_y}{E_{y0}} cos(\delta) - E_{z0} \sqrt{1 - \left(\frac{E_y}{E_{y0}}\right)^2} sin(\delta)$$

$$E_z - E_{z0} \frac{E_y}{E_{y0}} cos(\delta) = E_{z0} \sqrt{1 - \left(\frac{E_y}{E_{y0}}\right)^2} sin(\delta)$$

$$E_z^2 - 2E_{z0} \frac{E_y E_z}{E_{y0}} cos(\delta) + \frac{E_y^2 E_{z0}^2}{E_{y0}^2} cos^2(\delta) = E_{z0}^2 sin^2(\delta) - \frac{E_y^2 E_{z0}^2}{E_{y0}^2} sin^2(\delta)$$

$$E_z^2 - 2E_{z0} \frac{E_y E_z}{E_{y0}} cos(\delta) + \frac{E_y^2 E_{z0}^2}{E_{y0}^2} \left(sin^2(\delta) + cos^2(\delta)\right) = E_{z0}^2 sin^2(\delta)$$

$$E_z^2 - 2E_{z0} \frac{E_y E_z}{E_{y0}} cos(\delta) + \frac{E_y^2 E_{z0}^2}{E_{y0}^2} = E_{z0}^2 sin^2(\delta)$$

Dividing the whole equation by E_{z0}^2

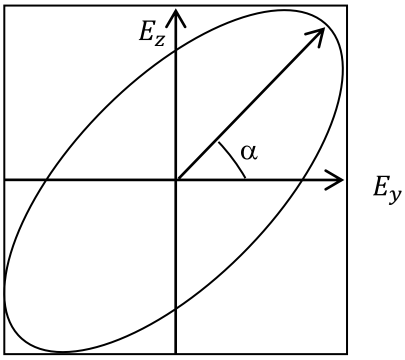
$$\frac{E_z^2}{E_{z0}^2} - 2 \frac{E_y E_z}{E_{y0} E_{z0}} cos(\delta) + \frac{E_y^2}{E_{y0}^2} = sin^2(\delta)$$

The above is a general equation of an ellipse. The major axis makes an angle α with axis along E_y. The tip of the resultant vector traces an ellipse in y-z plane.

$$tan(2\alpha) = E_{y0} E_{z0} \frac{cos(\delta)}{\left(E_{z0}^2 - E_{y0}^2\right)}$$

Case $\delta = 0, \pm 2n\pi$

$$\frac{E_y^2}{E_{y0}^2} - 2 \frac{E_y E_z}{E_{y0} E_{z0}} + \frac{E_y^2}{E_{y0}^2} = 0$$



$$\left(\frac{E_y}{E_{y0}} - \frac{E_z}{E_{z0}}\right)^2 = 0$$

$$E_y = \frac{E_{y0}}{E_{z0}} E_z$$

Straight line with a slope $\frac{E_{y0}}{E_{z0}}$

Case $\delta = \pi, \pm(2n+1)\pi$

$$\frac{E_z^2}{E_{z0}^2} + 2\frac{E_y E_z}{E_{y0} E_{z0}} + \frac{E_y^2}{E_{y0}^2} = 0$$

$$\left(\frac{E_y}{E_{y0}} + \frac{E_z}{E_{z0}}\right)^2 = 0$$

$$E_y = -\frac{E_{y0}}{E_{z0}} E_z$$

Straight line with a slope $-\frac{E_{y0}}{E_{z0}}$

Case $\delta = \frac{\pi}{2}, \pm(2n+1)\frac{\pi}{2}$

$$\frac{E_z^2}{E_{z0}^2} + \frac{E_y^2}{E_{y0}^2} = 1$$

Case $\delta = \frac{\pi}{2}, \pm(2n+1)\frac{\pi}{2}$ and $E_{y0} = E_{z0} = E_0$

Circularly polarized

$$E_x^2 + E_y^2 = E_0^2$$

Tutorial Questions

1. Explain the regular pattern due to heat treatment of windshields of cars.
2. What will be the Brewster's angle for a glass slab (n=1.5) immersed in water (n = 1.33).
3. Show that when a ray is incident at Brewster's angle the reflected ray is perpendicular to refracted ray.
4. If unpolarized light falls on a system two crossed polarized sheets, no light is transmitted. If a third polarizing sheet is placed between them will light be transmitted? Explain.
5. A ray of light strikes a glass plate at an angle of 60^0. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.
6. A beam of light is partially reflected and partially transmitted by a medium of refractive index $\sqrt{3}$. What is the polarizing angle of the medium.
7. No light passes through two perfect polarizing filters with perpendicular axes. However, if a third polarizing filter is placed between the original two, some light can pass. Why is this? Under what circumstances does most of the light pass?