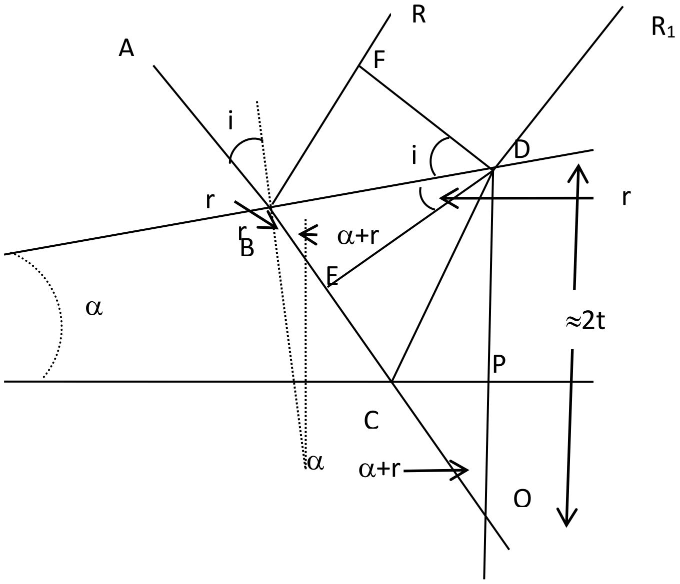
Wedge shaped thin film



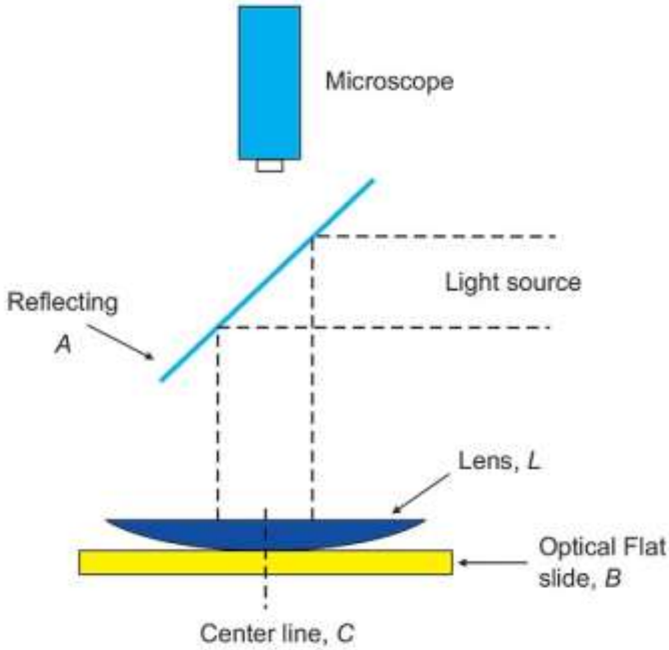
$$\delta = \frac{2\pi}{\lambda} 2\mu t cos(r+\alpha) + \pi = \begin{cases} 2n\pi & bright \\ (2n+1)\pi/2 & dark \end{cases}$$

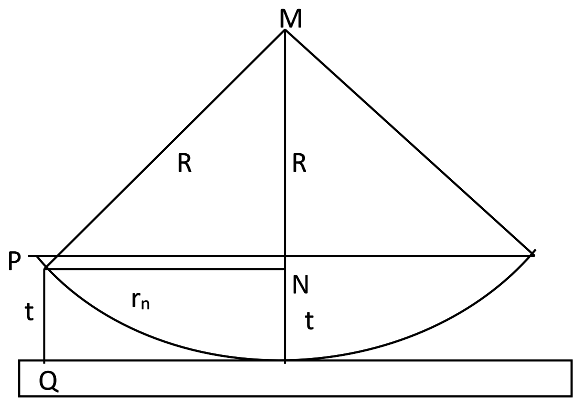
Condition

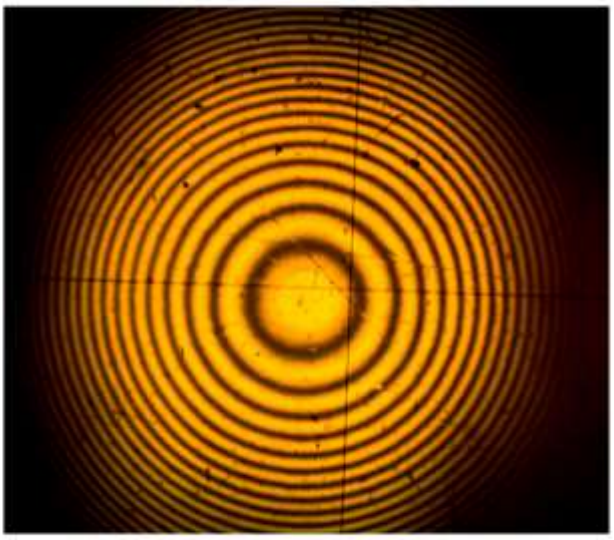
$$2\mu t cos(r+\alpha) = \begin{cases} (2n+1)\frac{\lambda}{2} & bright \\ n\lambda & dark \end{cases}$$

Newton's rings

A plano convex lens of large radius of curvature is placed on a plane glass plate. Light from monochromatic extended source is held parallel by a convex lens and is incident on a glass plate held at 45^0 to the incident film. The glass plate reflects the beam downward.


Microscope
Light source
Reflecting
A
Lens, L
Optical Flat
slide, B
Center line, C


M
R
R
P
N
r_n
t
t
Q



Part of the light is reflected from the curved surface of the lens and the transmitted part is reflected from the surface of the glass plate. This reflected part suffers a phase change of π.

These two reflected rays interfere and give rise to interference pattern in the form of circular rings.

The central ring is dark.

In order to remove the transmitted part and reflection from the lower surface of the glass plate the lower surface of the plate is blackened.

The diameter of the rings is proportional to square root of n. As ring number increases thickness of the ring decreases.

Central band will always be dark.

Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be OQ = r_m.

$$R^2 = r_n^2 + (R - t)^2$$

$$r_n^2 = 2Rt - t^2$$

As R >> t

$$r_n^2 \cong 2Rt$$

The condition

$$2\mu t cos(r) = \begin{cases} (2n+1)\frac{\lambda}{2} & bright \\ n\lambda & dark \end{cases}$$

When applied for bright fringes and normal incidence will give

$$2t = (2n+1)\frac{\lambda}{2}$$

Similarly for dark fringe

$$2t = n\lambda$$

Thus, the condition for dark fringe at Q is that

$$\frac{r_n^2}{R} \cong 2t = n\lambda$$

$$r_n^2 \cong Rn\lambda$$

Ring diameter

$$D_n \cong 2\sqrt{Rn\lambda}$$

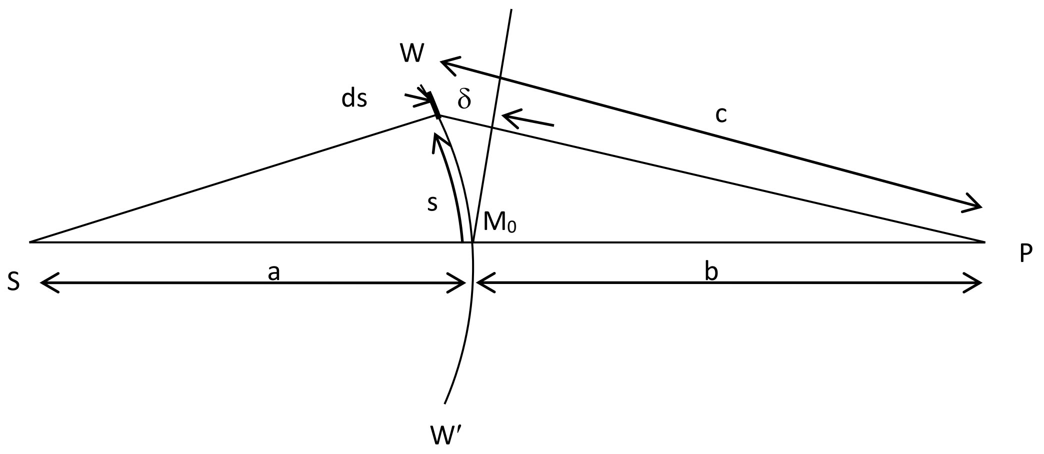
Determination of wavelength of light

$$D_n^2 \cong 4Rn\lambda$$

$$D_{n+p}^2 \cong 4R(n+p)\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Fresnel Diffraction at straight edge



Consider a wavefront WW′ from a source S. The field at P due to an elemental area ds at a distance s from the pole M_0. Let the electric field for all the points on the wavefront WW′ be$\boldsymbol{E_0}$

$$d\boldsymbol{E}_p = \frac{\boldsymbol{E_0}}{b}\sin(\omega t - k(b+\delta))\,ds$$

$$\boldsymbol{E}_p = \frac{\boldsymbol{E_0}}{b}\int \sin(\omega t - k(b+\delta))\,ds$$

$$\sin(\omega t - k(b+\delta)) = \sin(\omega t - kb)\cos(k\delta) - \cos(\omega t - kb)\sin(k\delta)$$

$$\boldsymbol{E}_p = \frac{\boldsymbol{E_0}}{b}\sin(\omega t - kb)\int \cos(k\delta)\,ds - \frac{\boldsymbol{E_0}}{b}\cos(\omega t - kb)\int \sin(k\delta) ds$$

Let

$$A\cos(\theta) = \int \cos(k\delta)\,ds$$

$$A\,sin(\theta) = \int \sin(k\delta) ds$$

$$\boldsymbol{E}_p = \frac{\boldsymbol{E_0}}{b}\sin(\omega t - kb)A\cos(\theta) - \frac{\boldsymbol{E_0}}{b}\cos(\omega t - kb)A\,sin(\theta)$$

$$\boldsymbol{E}_p = \frac{\boldsymbol{E_0}}{b}A\sin(\omega t - kb - \theta)$$

Intensity

$$\boldsymbol{I}_p = \boldsymbol{E_p^2} = \frac{\boldsymbol{E_0^2}}{b^2}A^2$$

Let

$$\boldsymbol{I}_p = \boldsymbol{E_p^2} = \frac{\boldsymbol{E_0^2}}{b^2}\left(\left[\int \cos(k\delta)\,ds\right]^2 + \left[\int \sin(k\delta)\,ds\right]^2\right)$$

$$\delta = \frac{a+b}{2ab}s^2$$

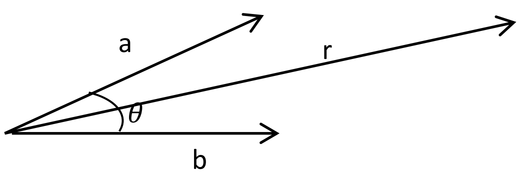
Proof:

$$\delta = c - b$$

$$c^2 = (a+b)^2 + a^2 - 2a(a+b)cos\theta$$

Identity

$$r^2 = a^2 + b^2 + 2abcos\theta$$



$$c^2 = (a+b)^2 + a^2 - 2a(a+b)\left(1 - \frac{\theta^2}{2!} + \cdots\right)$$

$$\approx a^2 + 2ab + b^2 + a^2 - 2a^2 - 2ab + 2a(a+b)\left(\frac{\theta^2}{2!}\right)$$

$$\approx b^2 + a(a+b)\theta^2$$

$$c^2 = b^2 + a(a+b)\left(\frac{s}{a}\right)^2$$

$$c^2 = b^2\left(1 + \frac{(a+b)}{a}\left(\frac{s}{a}\right)^2\right)$$

$$c \approx b\left(1 + \frac{(a+b)}{2ab^2}s^2\right)$$

$$\delta = c - b = \frac{(a+b)}{2ab}s^2$$

$$\boldsymbol{I}_p = \boldsymbol{E}_{\boldsymbol{p}}^{\boldsymbol{2}} = \frac{\boldsymbol{E}_{\boldsymbol{0}}^{\boldsymbol{2}}}{b^2}\left(\left[\int \cos\left(k\frac{(a+b)}{2ab}s^2\right)ds\right]^2 + \left[\int \sin\left(k\frac{(a+b)}{2ab}s^2\right)ds\right]^2\right)$$

$$\boldsymbol{I}_p = \frac{\boldsymbol{E}_{\boldsymbol{0}}^{\boldsymbol{2}}}{b^2}\left(\left[\int \cos\left(\frac{\pi}{\lambda}\frac{(a+b)}{ab}s^2\right)ds\right]^2 + \left[\int \sin\left(\frac{\pi}{\lambda}\frac{(a+b)}{ab}s^2\right)ds\right]^2\right)$$

Let

$$\frac{\pi}{\lambda}\frac{(a+b)}{ab}s^2 = \pi\frac{v^2}{2}$$

$$s = \sqrt{\frac{\lambda ab}{2(a+b)}}v$$

$$ds = \sqrt{\frac{\lambda ab}{2(a+b)}}dv$$

$$\boldsymbol{I}_p = \frac{\boldsymbol{E}_{\boldsymbol{0}}^{\boldsymbol{2}}}{b^2}\frac{\lambda ab}{2(a+b)}\left(\left[\int \cos\left(\pi\frac{v^2}{2}\right)dv\right]^2 + \left[\int \sin\left(\pi\frac{v^2}{2}\right)dv\right]^2\right)$$

$$\boldsymbol{I}_p = \kappa([\mathrm{C}(v)]^2 + [\mathrm{S}(v)]^2)$$

Fresnel integrals

$$\mathrm{C}(v) = \int_0^v \cos\left(\pi\frac{v^2}{2}\right)dv\,\mathrm{S}(v) = \int_0^v \sin\left(\pi\frac{v^2}{2}\right)dv$$

Properties

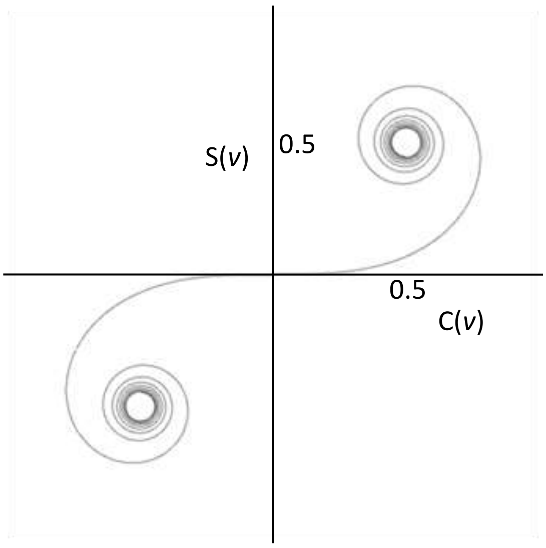
$$\lim_{v\to\infty}\int_0^v \cos\left(\pi\frac{v^2}{2}\right)dv = \frac{1}{2}\lim_{v\to\infty}\int_0^v \sin\left(\pi\frac{v^2}{2}\right)dv = \frac{1}{2}$$

$$\mathrm{C}(0) = 0 \qquad S(0) = 0$$

$$\mathrm{C}(\infty) = \frac{1}{2} \qquad S(\infty) = \frac{1}{2}$$

$$\mathrm{C}(-v) = -\,\mathrm{C}(v) \qquad S(-v) = -S(v)$$

Cornu Spiral



Intensity due to unobstructed wavefront

$$\boldsymbol{I}_0 = [\kappa([\mathrm{C}(v)]^2 + [\mathrm{S}(v)]^2)]_{-\infty}^{\infty}$$

Consider

Version 3.0

$$\mathrm{C}(v) = \int_{-\infty}^{\infty} \cos\left(\pi \frac{v^2}{2}\right) dv$$

$$\mathrm{C}(v) = \int_{-\infty}^{0} \cos\left(\pi \frac{v^2}{2}\right) dv + \int_{0}^{\infty} \cos\left(\pi \frac{v^2}{2}\right) dv$$

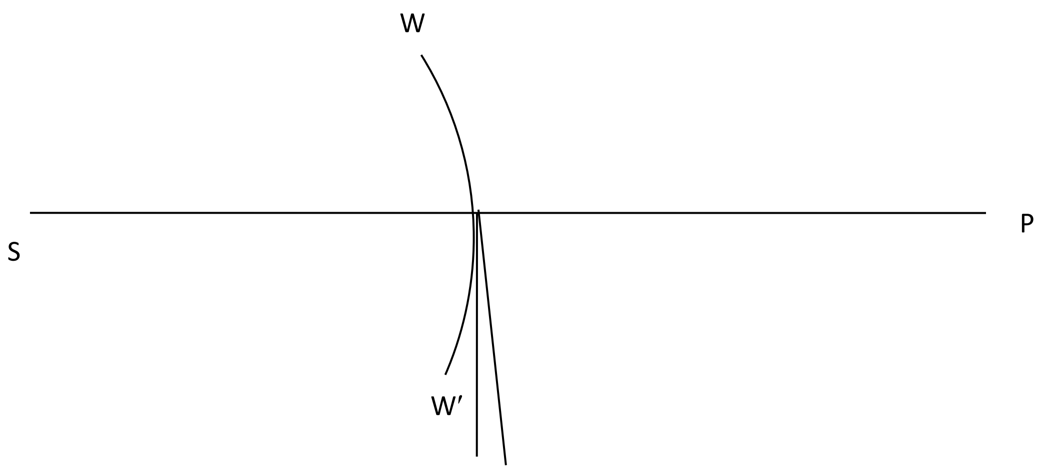
$$\mathrm{C}(v) = \frac{1}{2} + \frac{1}{2} = 1$$

$$\boldsymbol{I}_0 = \kappa(1+1) = 2\kappa$$

Thus

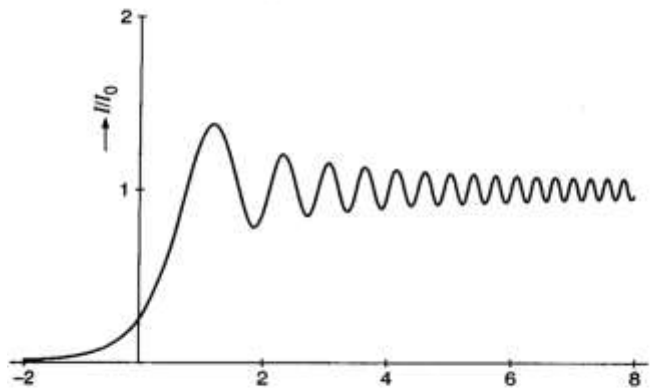
$$\boldsymbol{I}_p = \frac{\boldsymbol{I}_0}{2}([\mathrm{C}(v)]^2 + [\mathrm{S}(v)]^2)$$

Intensity due to straight edge



$$\boldsymbol{I}_{edge} = \left[\frac{\boldsymbol{I}_0}{2}([\mathrm{C}(v)]^2 + [\mathrm{S}(v)]^2)\right]_0^{\infty}$$

$$\boldsymbol{I}_{edge} = \frac{\boldsymbol{I}_0}{2}\left(\left[\frac{1}{2}\right]^2 + \left[\frac{1}{2}\right]^2\right) = \frac{\boldsymbol{I}_0}{4}$$


2
1
$\longrightarrow I/I_0$
-2
2
4
6
8