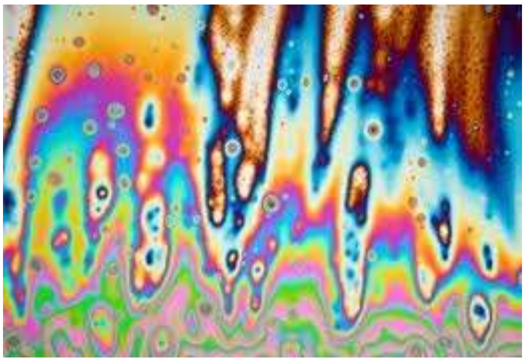
Unit-1

Interference

You must have observed this kind of pattern on oil films, soap bubbles etc.



Let us try to understand the reason for this pattern.

Light is an electromagnetic wave. The electric and magnetic fields are oscillating in space and time according the equation

$$\boldsymbol{E} = \boldsymbol{E_0}\sin(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)$$

The frequency of oscillations is so high $\frac{c}{\lambda} = 6 \times 10^{14}$Hz

$T = 1.6 \times 10^{-15}$s, that our eyes cannot resolve such time differences. What we/ or even a camera sees is a time average of the above equation.

Interference

Below is the simplified description given in most of the textbook. You may study this for the examination point of view, however, it may leave certain questions unanswered. A more detailed explanation for the unanswered questions is given below in the optional text below:

$$\boldsymbol{E_1} = \boldsymbol{E_{01}}\sin(\omega t)$$

$$\boldsymbol{E_2} = \boldsymbol{E_{02}}\sin(\omega t + \delta)$$

$$\boldsymbol{E} = \boldsymbol{E_{01}}\sin(\omega t) + \boldsymbol{E_{02}}\sin(\omega t + \delta)$$

$$\boldsymbol{E} = \boldsymbol{E_{01}}\sin(\omega t) + \boldsymbol{E_{02}}\sin(\omega t)\cos(\delta) + \boldsymbol{E_{02}}\cos(\omega t)\sin(\delta)$$

$$\boldsymbol{E} = (\boldsymbol{E_{01}} + \boldsymbol{E_{02}}\cos(\delta))\sin(\omega t) + \boldsymbol{E_{02}}\sin(\delta)\cos(\omega t)$$

Let

$$\boldsymbol{E_{01}} + \boldsymbol{E_{02}}\cos(\delta) = \boldsymbol{A}\cos(\varphi)$$

$$\boldsymbol{E_{02}}\sin(\delta) = \boldsymbol{A}\sin(\varphi)$$

$$\boldsymbol{E} = \boldsymbol{A}\cos(\varphi)\sin(\omega t) + \boldsymbol{A}\sin(\varphi)\cos(\omega t)$$

$$\boldsymbol{E} = \boldsymbol{A}\sin(\omega t + \varphi)$$

$$\boldsymbol{I} = < E^{2} > \cong \boldsymbol{A}^2$$

Average of a function in general is given by

$$< f(x) > = \frac{1}{b-a}\int_a^b f(x)dx$$

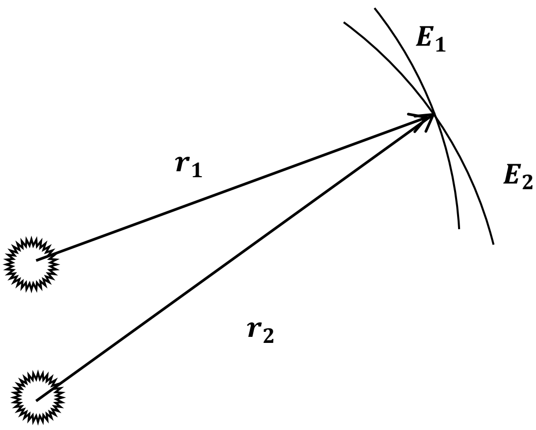
$$\boldsymbol{A}^2 = (\boldsymbol{E_{01}} + \boldsymbol{E_{02}}\cos(\delta))^{\mathbf{2}} + (\boldsymbol{E_{02}}\sin(\delta))^{\mathbf{2}}$$

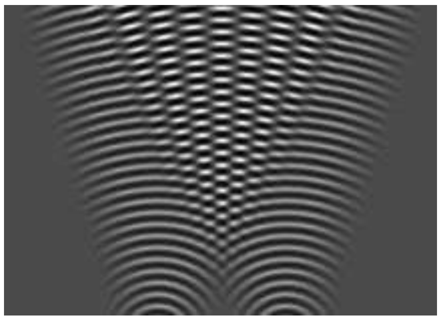
$$\delta = \boldsymbol{k_1}\cdot\boldsymbol{r_1} - \boldsymbol{k_2}\cdot\boldsymbol{r_2} + \boldsymbol{\epsilon_1} - \boldsymbol{\epsilon_2}$$

$$\boldsymbol{A}^2 = \left((\boldsymbol{E_{01}})^{\mathbf{2}} + 2\boldsymbol{E_{01}}\boldsymbol{E_{02}}\cos(\delta) + (\boldsymbol{E_{02}}\cos(\delta))^{\mathbf{2}}\right) + (\boldsymbol{E_{02}}\sin(\delta))^{\mathbf{2}}$$

$$I = (\boldsymbol{E_{01}})^{\mathbf{2}} + 2\boldsymbol{E_{01}}\boldsymbol{E_{02}}\cos(\delta) + (\boldsymbol{E_{02}})^{\mathbf{2}}$$

$$I = \boldsymbol{I_1} + \mathbf{2}\sqrt{\boldsymbol{I_1}\boldsymbol{I_2}}\cos(\delta) + \boldsymbol{I_2}$$





$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$$

For $I_0 = I_1 = I_2$

$$I = 2I_0(1 + \cos(\delta))$$

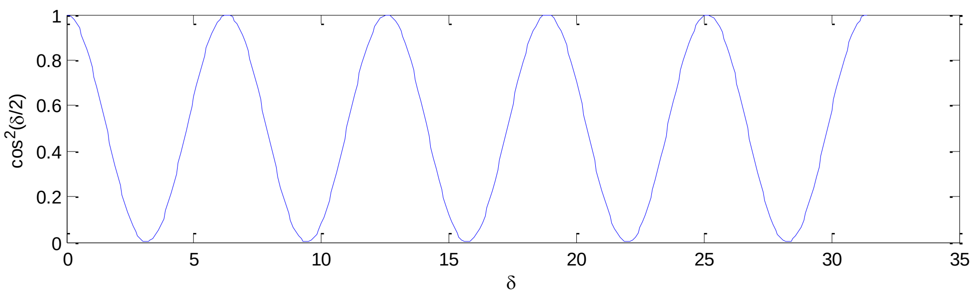
$$I = 4I_0\cos^2\left(\frac{\delta}{2}\right)$$

A plot of the above equation, and intensity profile is given below

$$\cos^2\left(\frac{\delta}{2}\right) = 1(\,bright\ fringe) \Longrightarrow \frac{\delta}{2} = n\pi$$

$$\cos^2\left(\frac{\delta}{2}\right) = 0(dark\ \ fringe) \Longrightarrow \frac{\delta}{2} = (2n+1)\frac{\pi}{2}$$

$$\delta = \begin{cases} 2n\pi & bright\ fringe \\ (2n+1)\pi dark\ fringe \end{cases}$$



Coherence

$$l_t = c\tau_0$$

$\Delta\nu$ represents frequency bandwidth

$$l_t = \frac{c}{\Delta\nu}$$

Since $\nu = \frac{c}{\lambda}$ thus $\Delta\nu = \left|-\left(\frac{c}{\lambda^2}\right)\Delta\lambda\right|$

$$l_t = \frac{\lambda^2}{\triangle\,\lambda}$$

Interference will be observed if the path difference is less than the coherence length of the incident light waves. Normally, the coherence length of the light from ordinary sources is of the order of a fraction of a millimeter. Therefore, the interference is seen with the path difference of the order of a few hundred microns only.

$$coherence\ lenght = \frac{\lambda^2}{\triangle\lambda}$$

$$2\mu t cos(r) < \frac{\lambda^2}{\triangle\lambda}$$

Examples:

White light	$\frac{550nm^2}{300nm} = 1000nm$
Green Light of Hg	$\frac{546nm^2}{0.025nm} = 1.2cm$
Krypton 86 isotope	$\frac{600nm^2}{0.00047nm} = 78cm$
CO_2 Laser	$\frac{10.6\mu m^2}{1\times10^{-5}nm} = 11km$

Multimode He-Ne lasers have a typical coherence length of 20cm, while coherence length of a single mode laser may exceed 100m. Single mode fiber laser can have coherence length of 100km.

Coherence time is the time over which the phase is predictable.

The condition of the maxima and minima for the above equation are

Phase difference ε_1-ε_2 should be constant for observable period of time.

Fringe Visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$I = 4I_0 \cos^2\left(\frac{\delta}{2}\right)$ gives $I_{max} = 4I_0$ and $I_{min} = 0$ thus V=1.

Consider a multilayer stack of alternating high-low index dielectric films. If each film has optical thickness of $\lambda/4$, all emerging beams are in phase. Multiple reflections in the region of λ increases the total reflected intensity and the quarter-wave stack performs as an efficient mirror. Such multilayer stacks can be designed to satisfy extinction or enhancement of reflected light over a greater portion of the spectrum than would a single-layer film.

Many useful and interesting applications of such multiplayer stack s of films are there.

Anti-reflecting multilayers in lenses

Display windows

Multipurpose broad and narrow band-pass filters

Thermal reflectors

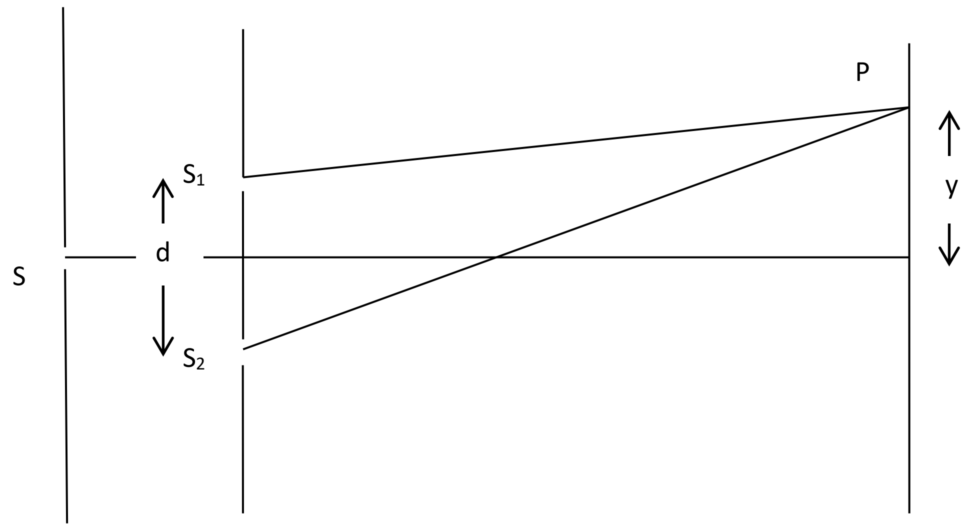
Cold mirrors used in projectors, dichroic mirrors in prismatic beam splitters

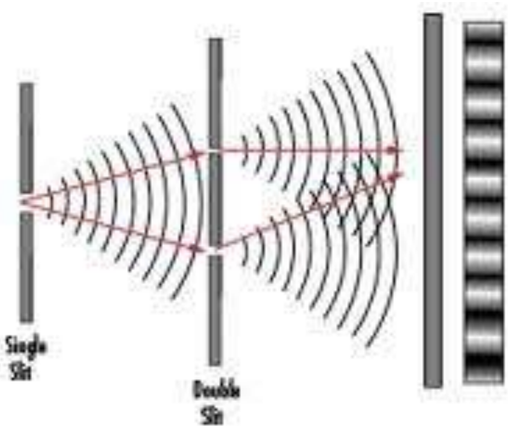
Dielectric mirrors used in gas lasers.

Human eye can detect change every 1/10th s but phase change of Na lamp occurs every 10^{-9}s.

Division of wave front	**Division of amplitude**
Beam is divided by allowing it to pass through passages of apertures placed side by side.	Phenomena of partial refraction and reflection are used to divide the light.
Useful with effectively small sources of light thus lesser intensity	Used with extended sources and thus effects may be of greater intensity.
Diffraction effects are present along with interference.	Diffraction effects are minimized
Examples: Young's double slit experiment Fresnel Biprism Fresnel double mirror	Dielectric films Michelson Interferometer Mach-Zehnder interferometer Newton's ring

Young's double slit experiment





$$\delta = \boldsymbol{k_1} \cdot \boldsymbol{r_1} - \boldsymbol{k_2} \cdot \boldsymbol{r_2} + \boldsymbol{\varepsilon_1} - \boldsymbol{\varepsilon_2}$$

When the two sources are derived from the same source then

$$\delta = \frac{2\pi}{\lambda}(\boldsymbol{r_1} - \boldsymbol{r_2})$$

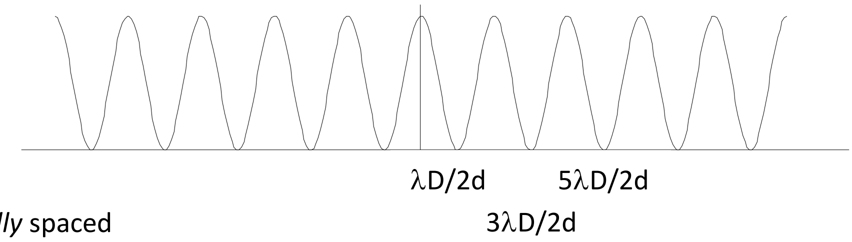
$$\delta = \frac{2\pi}{\lambda}(PS_1 - PS_2) = \frac{2\pi}{\lambda}\left(dsin(\theta)\right) = \frac{2\pi}{\lambda}\left(\frac{dy}{D}\right)$$

$$\delta = \frac{2\pi}{\lambda}\left(dsin(\theta)\right) = \begin{cases} 2n\pi & bright \\ (2n+1)\pi dark \end{cases}$$

$$path\ difference\ \ \Delta = dsin(\theta) = \begin{cases} n\lambda & bright \\ (2n+1)\frac{\lambda}{2} dark \end{cases}$$

$$I = 4I_0 cos^2\left(\frac{\delta}{2}\right)$$

$$I = 4I_0 cos^2\left(\frac{\pi}{\lambda}\left(\frac{dy}{D}\right)\right)$$



Fringes are *equally* spaced

Fringe width

n^{th} order bright fringe occurs when

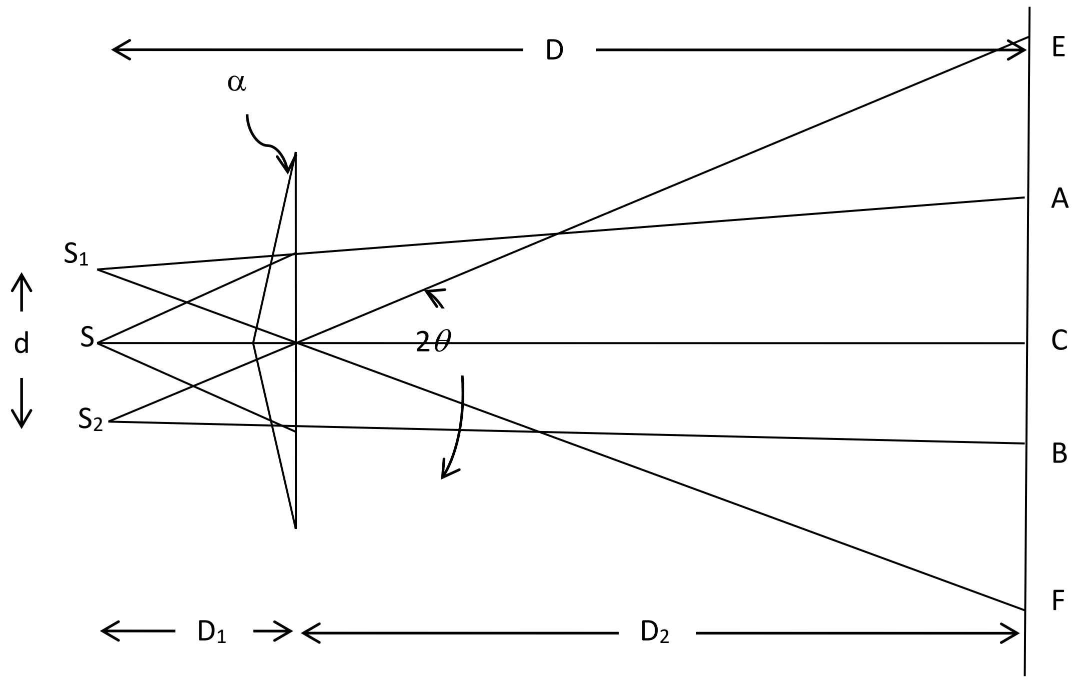
$$y = \frac{n\lambda D}{d}$$

(n+1) th order bright fringe occurs when

$$y = \frac{(n+1)\lambda D}{d}$$
$$\beta = \frac{\lambda D}{d}$$

$\frac{\lambda}{d}$ is called the angular width

Fresnel Biprism



Condition for Bright fringes

$$\frac{dy}{D_1 + D_2} = n\lambda$$

Condition for dark fringes

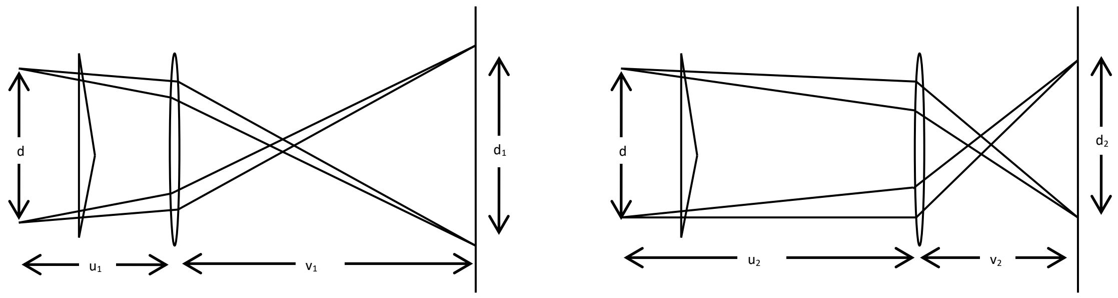
$$\frac{dy}{D_1 + D_2} = (2n+1)\frac{\lambda}{2}$$

Fringe width

$$\beta = \frac{\lambda(D_1 + D_2)}{d}$$

Double displacement method for measurement of '*d*'

A convex lens of focal length at least $\frac{(D_1+D_2)}{4}$ is placed between bi-prism and screen.



$$\frac{u_1}{v_1} = \frac{v_2}{u_2}$$

$$m_1 m_2 = 1$$

$$\frac{d_1}{d}\frac{d_2}{d} = 1$$

$$d = \sqrt{d_1 d_2}$$

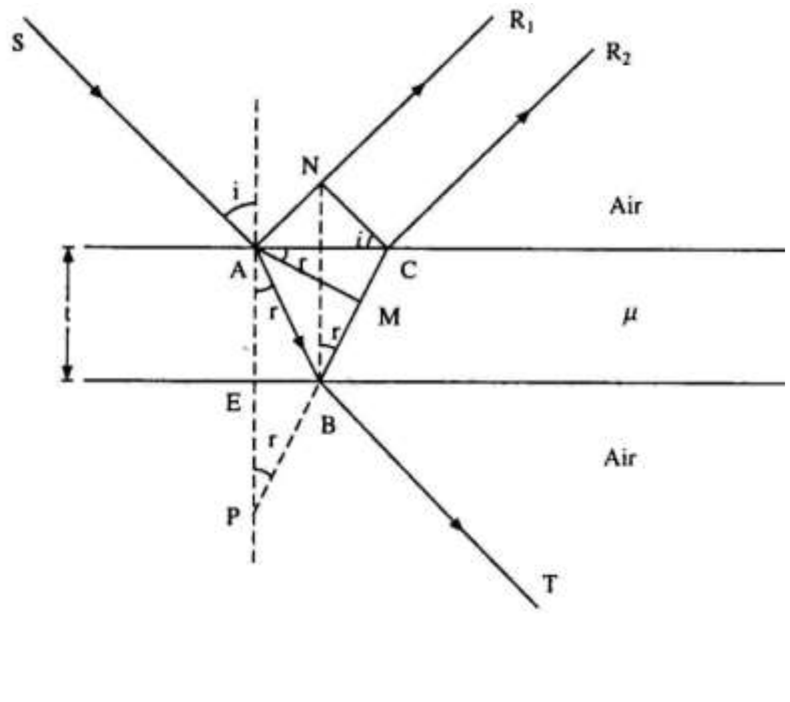
The distance d can also be determined by the following formula

$$d = 2a(\mu - 1)\alpha$$

Where μ is the refractive index of the material of the biprism, α is the small refracting angle of the prism, and a is the distance between the slit and the biprism.

Interference in thin films

Parallel films



$$Path\ difference\ =\ SR_1 - SR$$

$$= (AB + BC)\mu - AN$$

$$= (AB + BC)\mu - \mu CM$$

$$= (AB + BC - CM)\mu$$

$$= (AB + BM)\mu$$

$$= \mu PM$$

$$= 2\mu t cos(r)$$

Let μ>1 therefore AB suffers a phase change of π

$$\delta = \frac{2\pi}{\lambda}(path\ difference) +\ \epsilon_1 - \epsilon_2$$

$$\delta = \frac{2\pi}{\lambda} 2\mu t cos(r) + \pi = \begin{cases} 2\pi n \quad bright \\ (2n+1)\ \pi\ \ dark \end{cases}$$

Condition

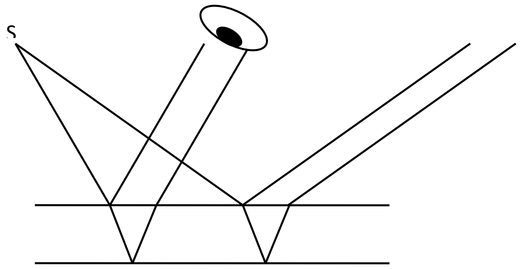
$$2\mu t cos(r) = \begin{cases} (2n+1)\frac{\lambda}{2} \quad bright \\ n\lambda \qquad\qquad dark \end{cases}$$

The intensities will not be the same because the interfering beams will have different amplitudes depending upon the light reflected and transmitted from the films.

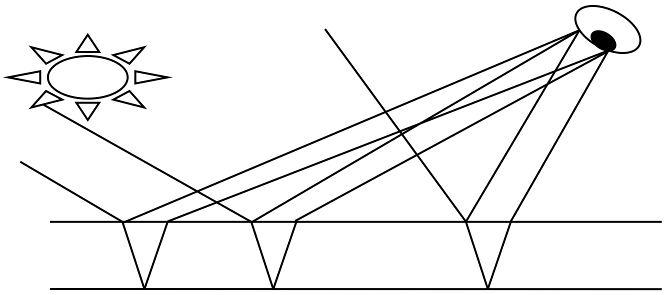
When a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is mλ , will be absent from the reflected light. The other colours will be reflected. Therefore the film will appear uniformly coloured with one colour being absent.

Requirement of extended source for viewing interference fringes in thin films

A broad source of light is required because a narrow source limits the area of the film that can be viewed



Extended Source



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Anti-reflection coatings

An anti-reflection coating eliminates reflected light and maximizes transmitted light in an optical system. The film is designed such that reflected light produces destructive interference and transmitted light produces constructive interference for a given wavelength of light.

The refractive index of the film is greater than the index of air and less than the index of glass

$$n_{air} < n_{coating} < n_{glass}$$

Tapetum Lucidum

A thin layer of the tissue called Tapetum Lucidum is a layer of tissue in the eye of many vertebrates. Lying immediately behind the retina, it is a retroreflector. It reflects visible light back through the retina, increasing the light available to the photoreceptors. The tapetum lucidum contributes to the superior night vision of some animals. Many of these animals are nocturnal. Presence of a tapetum lucidum enables animals to see in dimmer light than would otherwise be possible. The tapetum lucidum, which is iridescent, reflects light roughly on the interference principles. In the cat, the tapetum lucidum increases the sensitivity of vision by 44%, allowing the cat to see light that is imperceptible to human eyes.

Iridescence, in animals like peacocks

The bright, changing colors were not caused by dyes or pigments, but by microscopic structures in the form of thin films.